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A Class of Patterns Generated by Modification of Beenker's Pattern

BY T. SOMA AND Y. WATANABE

The Institute of Physical and Chemical Research, Wako-shi, Saitama 351-01, *Japan*

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Abstract

A modification of Beenker's pattern is considered that is generated by the transformation matrix obtained by applying the rotation matrix to that for Beenker's pattern. The symmetry of the modified pattern is discussed based on the transformation matrix. It is well known that Beenker's pattern, a two-dimensional eightfold quasiperiodic pattern, is characterized by the transformation matrix, the column vectors of which are the projected basis vectors in four-dimensional cubic lattice space.

1. Introduction

The theory of quasiperiodic patterns has been extensively studied in connection with the modeling of quasicrystals (Bak & Goldman, 1988). Among many methods of generating quasiperiodic patterns, the projection method is a standard and widely used method (Bak & Goldman, 1988). In this paper a two-dimensional eightfold symmetric quasiperiodic pattern or tiling called Beenker's pattern is reviewed first. It is generated by the projection method from the four-dimensional cubic lattice to the two-dimensional pattern space so as to have eightfold symmetry. Then, the modification of this pattern is considered by introducing an orthogonal transformation matrix based on the rotation in four-dimensional space. The rotation of the transformation matrix defining the pattern and test spaces (see § 2) has been considered by Kramer (1987) in connection with icosahedral and cubic symmetries. The phason strain, another kind of modification or deformation, with respect to Beenker's pattern is treated by Wang & Kuo (1988) and Socolar (1989).

2. A two-dimensional eightfold symmetric quasiperiodic pattern

It is known that a two-dimensional eightfold symmetric quasiperiodic pattern, Beenker's pattern (Beenker, 1982) is characterized by the orthogonal transformation matrix A (Wang & Kuo, 1988; Socolar, 1989; Soma, Watanabe & Ito, 1990),

$$A=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1/\sqrt{2} & 0 & -1/\sqrt{2}\\ 0 & 1/\sqrt{2} & 1 & 1/\sqrt{2}\\ 1 & -1/\sqrt{2} & 0 & 1/\sqrt{2}\\ 0 & 1/\sqrt{2} & -1 & 1/\sqrt{2}\end{pmatrix}. \qquad (1)$$

The column vectors correspond to basis vectors of the original axes (x_i) with respect to the transformed axes (x'_j). Since the upper and the lower two rows correspond to the pattern (parallel) and the test (perpendicular) space, respectively (Soma, Watanabe & Ito, 1990), the upper and lower two-dimensional column vectors $\mathbf{a}_i^{\parallel}$ and $\mathbf{a}_j^{\perp}$ ($i, j = 1, 2, 3, 4$) represent the projected basis vectors in their respective spaces as shown in Fig. 1. It is known that the pattern consists of a square and a rhombus of equal-length sides, as shown in Fig. 2. The pattern is thought of as a mixture of two square lattices rotated relative to each other by $\pi/4$. It is easy to see that the matrix A is generated by the product of four simple rotation matrices in four-dimensional space,

$$A = R_{13}(\alpha_{13})R_{24}(\alpha_{24})R_{34}(\alpha_{34})R_{23}(\alpha_{23}), \qquad (2)$$

with $\alpha_{13} = -\pi/4$, $\alpha_{24} = \pi/4$, $\alpha_{34} = \pi/4$ and $\alpha_{23} = \pi/2$, where $R_{ij}(\alpha_{ij})$ is the matrix representing a simple rotation in the x_ix_j plane by an angle α_{ij} from the axis x_i toward the x_j axis, such as

$$R_{12}(\alpha_{12})=\begin{pmatrix}\cos\alpha_{12} & \sin\alpha_{12} & 0 & 0\\ -\sin\alpha_{12} & \cos\alpha_{12} & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}. \qquad (3)$$

As is discussed by Wang & Kuo (1988), the pattern generated by the transformation matrix A has sym-

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metry D_{8h}, having three point symmetric operations; an eightfold rotation C_8, a vertical inversion v and a central inversion σ. The action of the element C_8 is derived as follows: consider a double rotation matrix $R(\theta, \varphi)$ with the angles θ and φ in the pattern and test spaces, respectively,

$$R(\theta,\varphi)=\begin{pmatrix}\cos\theta & \sin\theta & 0 & 0\\ -\sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & \cos\varphi & \sin\varphi\\ 0 & 0 & -\sin\varphi & \cos\varphi\end{pmatrix}. \quad (4)$$

It has been shown that (Weigel, Veysseyre, Phan, Effantin & Billiet, 1984; Veysseyre, Weigel, Phan & Effantin, 1984; Whittaker & Whittaker, 1986; Ishihara & Yamamoto, 1988) the action $\Gamma(C_8)$ of the element C_8 is $\tilde{A}R(\theta, \varphi)A$, with $\theta = \pi/4$ and $\varphi = 3\pi/4$ being respectively the angles between $\mathbf{a}_i^{\parallel}$ and $\mathbf{a}_{i+1}^{\parallel}$ and $\mathbf{a}_i^{\perp}$ and $\mathbf{a}_{i+1}^{\perp}$ $(i = 1, 2, 3)$, as shown in Fig. 1, and thus

$$\Gamma(C_8)=\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -1 & 0 & 0 & 0\end{pmatrix}, \quad (5)$$

which indicates eightfold rotational symmetry.

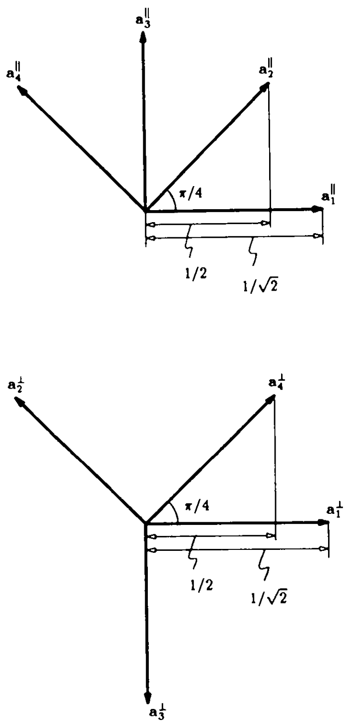

Fig. 1. Basis vectors in pattern and test space projected by (1).

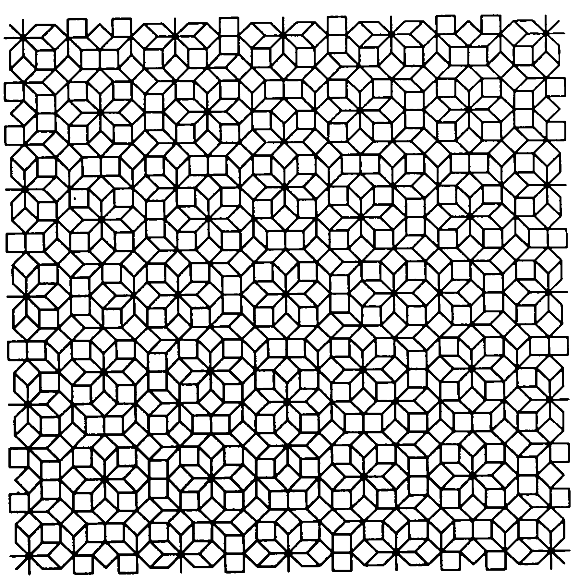

Fig. 2. Eightfold symmetric quasiperiodic pattern (Beenker's pattern).

3. Modification of Beenker's pattern

The first modification is a pattern that can be thought of as a mixture of two square lattices rotated relative to each other by an arbitrary angle α such that the x_1 component of $\mathbf{a}_2^{\parallel}$ is $(1+\delta)/2$ as shown in Fig. 3.

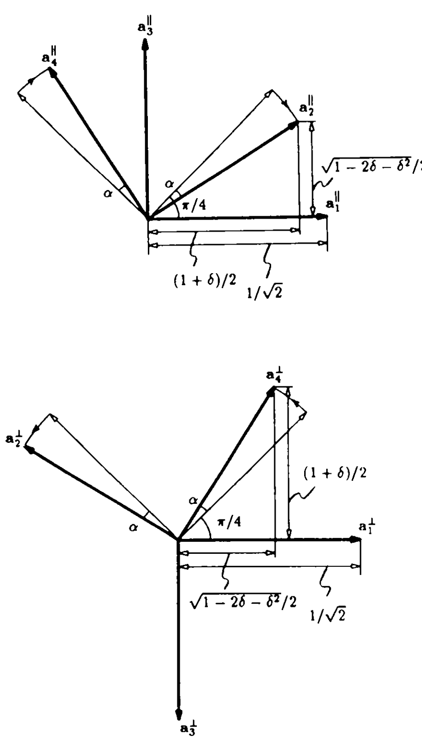

Fig. 3. Basis vectors in pattern and test space projected by (6).

The transformation matrix A_1 is given as

$$A_1(\delta)=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & (1+\delta)/\sqrt{2} & 0 & -\sqrt{1-2\delta-\delta^2}/\sqrt{2} \\ 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & 1 & (1+\delta)/\sqrt{2} \\ 1 & -(1+\delta)/\sqrt{2} & 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} \\ 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & -1 & (1+\delta)/\sqrt{2} \end{pmatrix}. \quad (6)$$

The projection matrices $P_1^{\parallel}$ and $P_1^{\perp}$ (Elser, 1986) when multiplied by A_1 on the right produce the matrices $A_1^{\parallel}$ and $A_1^{\perp}$,

$$A_1^{\parallel}=A_1P_1^{\parallel} \quad \text{and} \quad A_1^{\perp}=A_1P_1^{\perp}, \quad (7)$$

which have respectively the upper and lower two rows of A_1 and zeros in other rows (Soma, Wanatanbe & Ito, 1990). From (7) it is easy to derive the projection matrices $P_1^{\parallel}$ and $P_1^{\perp}$ for the pattern and the test space,

$$P_1^{\parallel}(\delta)=\frac{1}{2}\begin{pmatrix} 1 & (1+\delta)/\sqrt{2} & 0 & -\sqrt{1-2\delta-\delta^2}/\sqrt{2} \\ (1+\delta)/\sqrt{2} & 1 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & 0 \\ 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & 1 & (1+\delta)/\sqrt{2} \\ -\sqrt{1-2\delta-\delta^2}/\sqrt{2} & 0 & (1+\delta)/\sqrt{2} & 1 \end{pmatrix} \quad (8)$$

and

$$P_1^{\perp}(\delta)=I-P_1^{\parallel}(\delta)$$
$$=\frac{1}{2}\begin{pmatrix} 1 & -(1+\delta)/\sqrt{2} & 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} \\ -(1+\delta)/\sqrt{2} & 1 & -\sqrt{1-2\delta-\delta^2}/\sqrt{2} & 0 \\ 0 & -\sqrt{1-2\delta-\delta^2}/\sqrt{2} & 1 & -(1+\delta)/\sqrt{2} \\ \sqrt{1-2\delta-\delta^2}/\sqrt{2} & 0 & -(1+\delta)/\sqrt{2} & 1 \end{pmatrix}, \quad (9)$$

where I is the unit matrix in four-dimensional space. It can be shown that A_1 is derived by multiplying A on the left by the rotation matrix $AR_{24}(\alpha_{24})A^{-1}$ and it is decomposed into four simple rotation matrices,

$$A_1(\alpha_{24})=R_{14}(\alpha_{24}/2)R_{23}(\alpha_{24}/2) \times R_{12}(\alpha_{24}/2)R_{34}(-\alpha_{24}/2)A, \quad (10)$$

with $1+\delta=\cos\alpha_{24}+\sin\alpha_{24}$. As a special case, put $\delta=(\sqrt{3}-\sqrt{2})/\sqrt{2}$, then the matrix A_1 becomes

$$A_1[\delta=(\sqrt{3}-\sqrt{2})/\sqrt{2}]=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1/2 & 1 & \sqrt{3}/2 \\ 1 & -\sqrt{3}/2 & 0 & 1/2 \\ 0 & 1/2 & -1 & \sqrt{3}/2 \end{pmatrix} \quad (11)$$

and the projection matrices $P_1^{\parallel}$ and $P_1^{\perp}$ become

$$P_1^{\parallel}[\delta=(\sqrt{3}-\sqrt{2})/\sqrt{2}]=\frac{1}{2}\begin{pmatrix} 1 & \sqrt{3}/2 & 0 & -1/2 \\ \sqrt{3}/2 & 1 & 1/2 & 0 \\ 0 & 1/2 & 1 & \sqrt{3}/2 \\ -1/2 & 0 & \sqrt{3}/2 & 1 \end{pmatrix} \quad (12)$$

and

$$P_1^{\perp}[\delta=(\sqrt{3}-\sqrt{2})/\sqrt{2}]=\frac{1}{2}\begin{pmatrix} 1 & -\sqrt{3}/2 & 0 & 1/2 \\ -\sqrt{3}/2 & 1 & -1/2 & 0 \\ 0 & -1/2 & 1 & -\sqrt{3}/2 \\ 1/2 & 0 & -\sqrt{3}/2 & 1 \end{pmatrix}. \quad (13)$$

The pattern is shown in Fig. 4; it consists of three types of rhombic tiles and is quasiperiodic, having fourfold symmetry. The action of the rotational element of the point symmetric operation is obtained in the same way as above; $\tilde{A}_1R(\theta,\varphi)A_1$, with $\theta=\pi/2$ and $\varphi=-\pi/2$ being respectively the angles between $\mathbf{a}_1^{\parallel}$ and $\mathbf{a}_3^{\parallel}$ or $\mathbf{a}_2^{\parallel}$ and $\mathbf{a}_4^{\parallel}$, and $\mathbf{a}_1^{\perp}$ and $\mathbf{a}_3^{\perp}$ or $\mathbf{a}_2^{\perp}$ and $\mathbf{a}_4^{\perp}$,

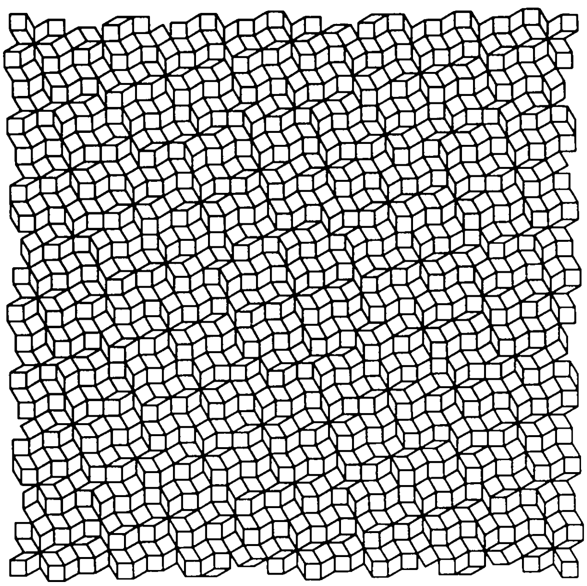

Fig. 4. Modified Beenker pattern generated by (11).

which can be transformed into the standard form as

$$\Gamma(C_4)=\begin{pmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{pmatrix}, \tag{14}$$

showing the fourfold rotational symmetry.

Next consider the pattern generated by modifying the length but fixing the orientations of basis vectors $\mathbf{a}_i^{\parallel}$ as shown in Fig. 5. The transformation matrix A_2 is

$$A_2(\delta)=\frac{1}{\sqrt{2}}\begin{pmatrix}(1+\delta) & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & 0 & -\sqrt{1-2\delta-\delta^2}/\sqrt{2}\\ 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & (1+\delta) & \sqrt{1-2\delta-\delta^2}/\sqrt{2}\\ \sqrt{1-2\delta-\delta^2} & -(1+\delta)/\sqrt{2} & 0 & (1+\delta)/\sqrt{2}\\ 0 & (1+\delta)/\sqrt{2} & -\sqrt{1-2\delta-\delta^2} & (1+\delta)/\sqrt{2}\end{pmatrix}. \tag{15}$$

It can be shown that A_2 is derived by multiplying A on the left by the rotation matrices R_{13} and R_{24},

$$A_2(\alpha)=R_{13}(\alpha_{13})R_{24}(\alpha_{24})A, \tag{16}$$

with $1+\delta=\cos\alpha+\sin\alpha$, where $\alpha=\alpha_{13}=-\alpha_{24}$. As an example, take the case of $\delta=(\sqrt{2}-\sqrt{3})/\sqrt{3}$; the transformation matrix becomes

$$A_2[\delta=(\sqrt{2}-\sqrt{3})/\sqrt{3}]=\frac{1}{\sqrt{6}}\begin{pmatrix}\sqrt{2}&\sqrt{2}&0&-\sqrt{2}\\0&\sqrt{2}&\sqrt{2}&\sqrt{2}\\2&-1&0&1\\1&0&-2&1\end{pmatrix} \tag{17}$$

and the pattern is periodic in both the x_1 and the x_2 directions as shown in Fig. 6.

The periodicity is known from the fact that the component ratio of the lower two row vectors in (17) is rational. Since these vectors are used to generate the coordinates of the projected point in the test space by multiplying the lattice vector in four-dimensional space, both the x_3 and the x_4 coordinates are discretized with the step $1/\sqrt{6}$, which shows the periodicity in the x_1 and x_2 directions. The period is found by considering the minimum loop in the test space consisting of $\mathbf{a}_1^{\perp}$, $\mathbf{a}_2^{\perp}$ and $\mathbf{a}_4^{\perp}$ for the x_1 direction and $\mathbf{a}_3^{\perp}$, $\mathbf{a}_2^{\perp}$ and $\mathbf{a}_4^{\perp}$ for the x_2 direction, respectively. They are $(\mathbf{a}_2^{\perp}, -\mathbf{a}_4^{\perp}, \mathbf{a}_1^{\perp})$ and $(\mathbf{a}_3^{\perp}, \mathbf{a}_4^{\perp}, \mathbf{a}_2^{\perp})$. The corresponding vector sequences in the pattern space are $(\mathbf{a}_2^{\parallel}, -\mathbf{a}_4^{\parallel}, \mathbf{a}_1^{\parallel})$ and $(\mathbf{a}_3^{\parallel}, \mathbf{a}_4^{\parallel}, \mathbf{a}_2^{\parallel})$, from which the period is shown to be $\sqrt{3}$ in both the x_1 and the x_2 directions (the details will be discussed elsewhere).

The action of the rotational element of the point symmetric operation in this case is shown to be $\tilde{A}_2R(\theta,\varphi)A_2$ with $\theta=0$ for $\mathbf{a}_4^{\parallel}$ and $\varphi=\pi$ for $\mathbf{a}_4^{\perp}$, which can be transformed into the standard form,

$$\Gamma(v)=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}, \tag{18}$$

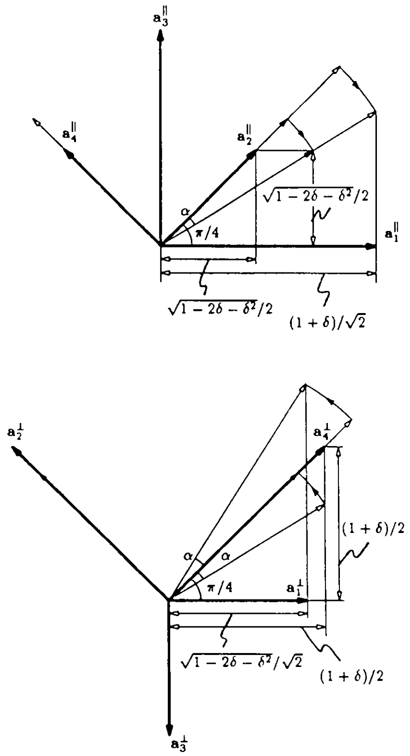

Fig. 5. Basis vectors in pattern and test space projected by (15).

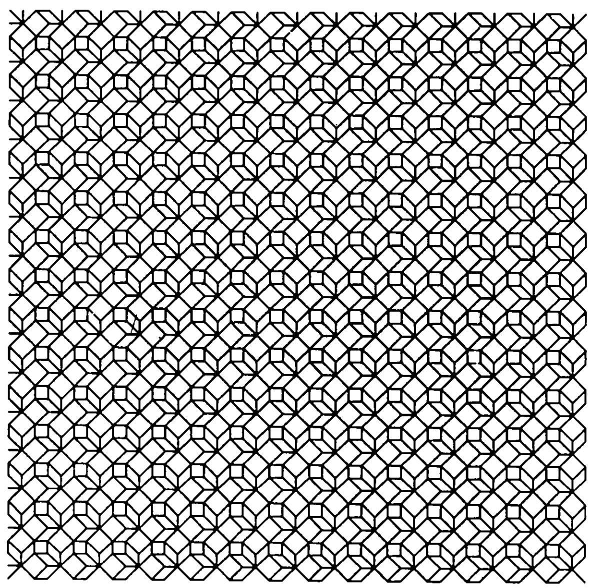

Fig. 6. Modified Beenker pattern generated by (17).

showing that the pattern has mirror symmetry, in which $\mathbf{a}_4^{\parallel}$ is the mirror axis.

Now, consider the pattern which is periodic in the x_2 direction but quasiperiodic in the x_1 direction. By modifying the x_2 component of the basis vectors in the pattern space by the amount δ (Fig. 7), one obtains the transformation matrix A_3,

$$A_3(\delta)=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & (1+\delta)/\sqrt{2} & \sqrt{1-2\delta-\delta^2} & (1+\delta)/\sqrt{2} \\ 1 & -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & \sqrt{1-2\delta-\delta^2}/\sqrt{2} & -(1+\delta) & \sqrt{1-2\delta-\delta^2}/\sqrt{2} \end{pmatrix}. \quad (19)$$

It can be shown that A_3 is derived by multiplying A on the left by the rotation matrix R_{24},

$$A_3(\alpha_{24})=R_{24}(\alpha_{24})A, \quad (20)$$

with $1+\delta=\cos\alpha_{24}+\sin\alpha_{24}$. As an example, take $\delta=(2-\sqrt{3})/\sqrt{3}$; the transformation matrix becomes

$$A_3[\delta=(2-\sqrt{3})/\sqrt{3}]=\frac{1}{\sqrt{6}}\begin{pmatrix} \sqrt{3} & \sqrt{3}/\sqrt{2} & 0 & -\sqrt{3}/\sqrt{2} \\ 0 & \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3}/\sqrt{2} & 0 & \sqrt{3}/\sqrt{2} \\ 0 & 1 & -2 & 1 \end{pmatrix} \quad (21)$$

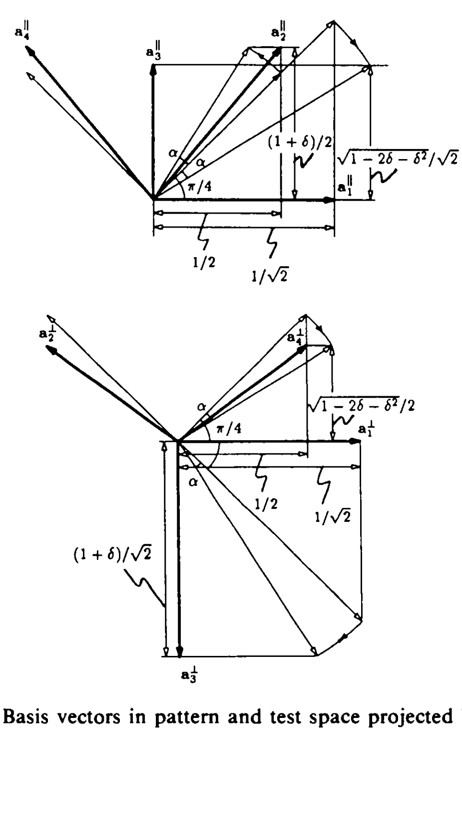

Fig. 7. Basis vectors in pattern and test space projected by (19).

and the pattern is shown in Fig. 8, which is periodic in the x_2 direction and quasiperiodic in the x_1 direction, the period being $\sqrt{3}$, which is shown by considering the minimum loop in the test space and the corresponding vector sequence in the pattern space, as in the previous case.

The action of the rotational element of the point symmetric operation is the same as the A_2 case with a mirror axis of $\mathbf{a}_3^{\parallel}$. The angles $\theta=0$ for $\mathbf{a}_3^{\parallel}$ and $\varphi=-\pi$ for $\mathbf{a}_3^{\perp}$ lead to $\Gamma(v)$ as given in (18).

All the transformation matrices considered above are orthonormal and can be derived by the product of simple rotation matrices. In three-dimensional space, the Euler-angle parametrization of the orthogonal matrix is known and can be derived by the product of three simple rotation matrices with different angle parameters $[R_{12}(\psi)R_{13}(-\theta)R_{12}(\varphi)]$. The generalization to four-dimensional space is straightforward and can be shown to be

$$R_{12}(\alpha_{12})R_{13}(\alpha_{13})R_{14}(\alpha_{14})\ldots R_{34}(\alpha_{34}). \quad (22)$$

There are six parameters in total, corresponding to those necessary to specify the rotated axes relative to the original axes. The number of parameters is the same as the number of ways of choosing two items from four and equals the number of independent planes of rotation. All the above transformation matrices are derived by assigning appropriate values to these parameters.

4. Infinitesimal rotation

In the preceding discussions, the parameter δ can take an arbitrary value, but if δ is restricted to an infinitesimal value, another class of modifications can

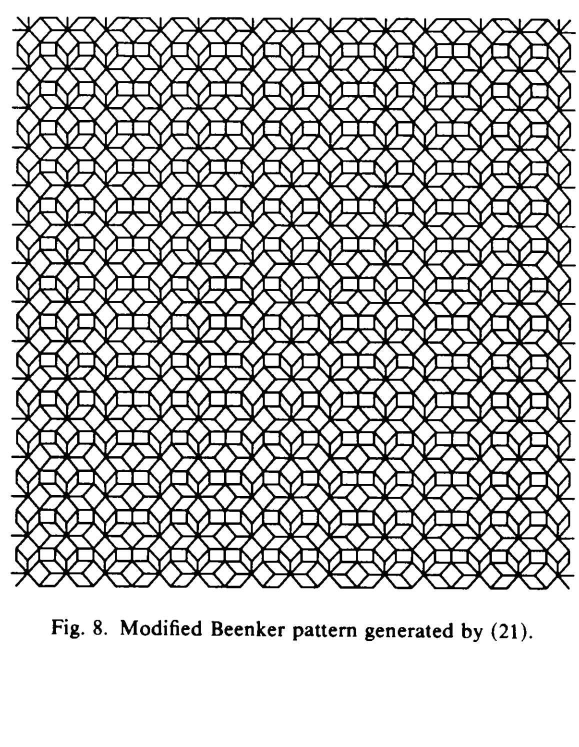

Fig. 8. Modified Beenker pattern generated by (21).

be examined in which the transformation matrix is only approximately orthogonal. A typical example is a transformation matrix in which only the test space part is modified so that the periodic pattern can be generated as an approximant to the Beenker pattern as well as the quasiperiodic one. A transformation matrix $A_4(\varepsilon)$ satisfying this condition is

$$A_4(\varepsilon)=\frac{1}{2}\begin{pmatrix} \sqrt{2} & 1 & 0 & -1 \\ 0 & 1 & \sqrt{2} & 1 \\ \sqrt{2} & -\sqrt{1+\varepsilon} & \varepsilon\sqrt{2}/2 & \sqrt{1+\varepsilon}/(1+\varepsilon) \\ \varepsilon\sqrt{2}/2 & \sqrt{1+\varepsilon}/(1+\varepsilon) & -\sqrt{2} & \sqrt{1+\varepsilon} \end{pmatrix}, \qquad (23)$$

which is orthonormal, neglecting the terms of $O(\varepsilon^2)$. The projected basis vectors in the test space are obtained by rotating that for the original Beenker pattern anticlockwise by an angle $\varepsilon/2$. For $\varepsilon=1/49$, A_4 becomes

$$A_4(\varepsilon=1/49)=\frac{\sqrt{2}}{980}\begin{pmatrix} 490 & 490/\sqrt{2} & 0 & -490/\sqrt{2} \\ 0 & 490/\sqrt{2} & 490 & 490/\sqrt{2} \\ 490 & -350 & 5 & 343 \\ 5 & 343 & -490 & 350 \end{pmatrix}, \qquad (24)$$

which is shown to be periodic with minor period $(70+99/\sqrt{2})$ in both the x_1 and the x_2 directions (the details will be discussed elsewhere).

5. Concluding remarks

A class of patterns is considered that is generated by applying rotation matrices to the transformation matrix corresponding to Beenker's pattern; some are quasiperiodic and some are periodic in one direction or in both the x_1 and the x_2 directions. The transformation matrix corresponding to these patterns can be specified by assigning appropriate values to angle parameters for the Euler-angle parametrization of the orthogonal transformation matrix in four-dimensional space, which has six independent parameters. The case in which only the test space part of the transformation matrix is modified is also considered. The three types of rhombic tiles in Fig. 4 and those in the 12-fold pattern obtained from a six-dimensional hypercubic lattice by projection (Ishihara, 1985; Gähler & Rhymer, 1986) suggest the existence of a continuous path of deformation, including the phason strain, between these patterns.

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